

The Twelve Answers Problem

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Why a grain boundary has one angle and twelve rotations, and what that costs a defect map

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ABSTRACT

A disorientation angle is one number. The rotation that produces it is not.

Reducing a misorientation by the crystal’s point group returns a unique minimum angle and a **non-unique rotation**: twelve equally-valid rotors for hexagonal 622, twenty-four for cubic 432. **None of that is news.** The multiplicity of symmetrically equivalent misorientations is textbook, and conventions for selecting a unique representative were published decades ago. Our claim is narrower: **the convention in everyday use specifies an angle, not a rotor**, and the gap between those two things has a measurable, platform-dependent cost.

The identity is harmless for any quantity built from angles. It is not harmless for any quantity built by **composing rotors around a loop**, because a loop multiplies rotations and therefore multiplies arbitrary choices.

We measure the cost on a public EBSD map of twinned magnesium. Between two machines running identical code on identical bytes, **453 plaquettes move by more than one degree, 201 change defect classification, and one cell reads 0.0000° on one platform and 146.6018° on the other.** Across five valid tie-break rules the boundary-seam count ranges **951 to 2207** — a factor of 2.3 from a bookkeeping choice. Grain-ring closures lose rank order entirely (Spearman 0.02) while moving a median of 67°.

Everything angle-based reproduces bit-exact across the same platforms.

We report the fix — exact-constant group elements, a quantised selection rule, and a record split into invariant and gauge tiers — verified on a third platform. We report two of our own results that the discipline killed: a striking spike at exactly 60° that was a tie-break artifact, and a grain-network defect census that was spanning-tree dependent. And we note that one established route, Bollmann’s O-lattice construction, is **structurally immune** because it never composes reduced rotors at all — a reason to prefer it that we have not seen stated.

A number that moves ten percentage points when you reorder a list of twelve matrices was never data.

Keywords: EBSD · electron backscatter diffraction · disorientation · symmetry reduction · crystallographic gauge · reproducibility · cross-platform determinism · disclination · triple junction · Frank vector · O-lattice · Bollmann · geometrically necessary dislocations · kernel average misorientation · Mackenzie distribution · noise floor · point group · quaternion · holonomy · metrology

Draft v1 — 2026-08-06. Everything measured here runs on one public file (`twins.ctf`, MTEX sample data, sha256 `cdb0e406...`) and one script. No proprietary data, no unpublished specimens.

SECTION 0 — WHY METALLURGY AT ALL

(Author’s voice. Provenance, not proof. Skip it if you want the results.)

This was the last stop.

I had a portable holonomy instrument and a methodology to go with it, and I had been walking it across substrates for months to find out where it worked and where it lied. Metallurgy was the final test loop — the Loop Walker closing its own circuit.

None of this was ever about money or a patent. The point was disaster precursors: if you can read a loop that fails to close, you might read a fault before it slips. So the instrument went to geodesy and weather first, and it worked. GPS vectors as strain gauges, Euler models of the seams, plate curvature read as a ternary — heptagon saddle at minus one, hexagon seam at zero, pentagon well at plus one. That was where crossing domains started looking like a method instead of a hobby.

Then the stars. Asteroseismology is hella noisy and wanted priors I did not have, so I looked for a quieter place to stand and tried reading the out-of-phase rim curl instead. Then I aimed at the noisiest quasars I could find, on the theory that a strong signal beats a weak one.

That was backwards, and finding out was the first thing this instrument taught me. **Holonomy likes the quiet.** It hides in plain sight. Point it at something loud and you measure the loudness. So I picked the quiet, non-moving quasars instead, and there it was. Curvature geometry loves cosmology — curvature points at dark matter, and by the end of that spoke I had a Gaia-driven curvature detector building Euler n-gon meshes and tilting their pose at cluster candidates.

That spoke ran against kill bars written before the data was touched, and it came back split: **85 of 509 rings at 7σ , parity-odd to 6×10^{-15}** — and two other bars failed cleanly with the mechanism named, and a third came back data-starved. Which is exactly the shape you want. It is also where I met my first competence surface: the map of where the tool works and where it does not, which turns out to be the only honest thing an instrument can hand you.

So I built recon spokes for the remaining domains. And I saved the slam dunks for last, which is exactly the wrong order, and they were the most painful education in the whole program.

Biology first. Knots, mirror math, and prions — which are just biological knots wearing a medical name. Chern-Simons is knots too, and my world lines and Wilson loops felt like they were coming home. DNA and RNA looked like they would have plenty of geometry to share — helical braiding, mirror braids more than knots. The results were fascinating and the existing tools were already good, and the thing that actually stopped us was **language**. Cross-domain words like *depth* and *moves* mean different things on either side of a fence, and the standards change under you while you are reading them. That is a barrier nobody warns you about and it cost more than any calculation.

So we stopped and consolidated. I wrote the methods paper — the substrates, the hooks, the traps — and published it to keep track, and to save the next loop walker the same weeks.

Which brings me here, and it is not a swerve. Metallurgy had been sitting at the back of this whole program for months, because of what the memory work needed.

Everything in this line is the natural evolution of the **Matriarch-Daughter Array**: an external field piloting passive motes by standing waves, built for undersea work and for in-body delivery-and-recovery, and scaled up to a framelocked array telescope that lives or dies on rad-hard memory. That architecture needs somewhere to *keep* things. The Crystal Family invariant-based topological memory work is the answer I had been sketching, and it is the reason metallurgy was ever on the list.

The whole stack is one sentence, and every clause is a loop walk. **The alphabet is the word. The word is the journey to the recording. The hypotenuse is the address and the direction vector.** The medium is glass because glass is cheap and everywhere. The engraving is done with vortex beams — lasers, microwaves, electron beams, all the same object at different wavelengths. And the winding you engrave *is* the physically unclonable function, because you cannot copy a topology by measuring an amplitude. The address is a loop walk and the stored ideogram is a loop walk, and they are the same kind of object, which is the part that took me longest to believe.

So no, I did not walk into metallurgy expecting to teach a metallurgist anything. I walked in hoping somebody in the field, smarter than me, might see the instrument and want to try it themselves. That was the whole ambition — get holonomy taken seriously by people who actually know the material.

What I found instead was that metallurgy is **spec-driven**. You write a spec, and everyone follows the spec. That is the read, and it turned out to matter more than anything I brought with me — because the thing that broke was not the material and not the detector. It was a convention inside the spec that nobody had thought to test.

A grain boundary has one angle and twelve rotations.

When that landed I did not think *finding*. I thought: here we go again, another illusion, another geometric ghost. Two days earlier a spike had shown up at exactly sixty degrees, looking beautiful and quantised, and it turned out to be the alphabetical order of a list of matrices. So the reflex by then was suspicion, not excitement.

That reflex is the paper. In a couple of days, four separate findings died to controls written before the runs: the sixty-degree spike was a tie-break, a defect census was a spanning tree, a numerical read was a discretisation artifact, and the audit target I had lined up evaporated the moment I read the actual source instead of a summary of it. **That did not cost anything. It taught me everything the paper knows.** Every one of those is in Section 9, by name, because a methods paper that shows only the clean case is a paper nobody should believe.

I came in with a grievance and I am leaving without one, and the trade was worth making.

The grievance was going to be about the eight-colour crayon box — that a field which genuinely needs new designs restricts its designers to a handful of sanctioned marks when there are watercolours, markers, lasers and pens. I thought I was visiting somebody's sandbox. What I did not understand is that they **own the sandbox, maintain it, and are responsible for every toy in it.**

Mass-produced memory is spec-driven because it has to be. The money is in formal, ubiquitous hardware and OEM manufacturing at consumer volume — a slightly cheaper, slightly more efficient chipset in the common socket. Not in a clever new detector. When ten thousand parts a shift have to be interchangeable, a spec is not a cage, it is the only thing that makes the industry possible at all. The eight colours *are* the spec points, and they are eight because eight is what survives being manufactured by everyone, everywhere, to the same tolerance.

So I cannot complain that they are making the wrong decisions. They are making the right ones for the mission they actually have.

What I found instead was a fit problem, and it is mine, not theirs. I brought months of work on invariant-based memory hoping to offer a tweak of value, and what I discovered is that **this was probably not the right home for it.** Not because the idea is wrong — because the idea asks you to bend a rule, and this is a field whose entire value proposition is that nobody bends the rules. Those ideas will have to find their own audience, and finding that out cost two days and was worth every hour.

What *does* belong here is much smaller, and it fits the industry's own format: **a spec.** One page, no new equipment, pass or fail. That is in Section 8, and it is the only thing in this paper I would ask a metallurgist to adopt.

Why did I find it and not someone in the field? Not because I know more. I know much less. It is that I kept asking whether a number moved when I was not looking — a habit from poker tables and Byzantine sensors, where every reading is somebody's claim and half of them are bluffing. Inside crystallography the tie-break is not a thing you are taught to see, because it is not a choice anybody made on purpose. It is just what the library returns.

So here is what I want from you, and it is one page.

Change your tie-break. Permute your symmetry operator list. Re-run. Diff.

Anything that moved is gauge. Publish it with the gauge named, or do not publish it.

That is the whole ask. It costs an afternoon, it needs nothing you do not already own, and it is written to drop into a spec without asking anyone to bend a rule.

The rest of what I brought — the memory work, the windings, the invariant you engrave rather than measure — does not belong in your spec and I am not going to pretend otherwise. It will have to find its own room. This page is the part that fits.

And if it holds still under every gauge you can think of, that is worth knowing too. A convention that survives being attacked is not a rule anymore. It is a result.

1. THE PROBLEM IN ONE PARAGRAPH

Two grains meet. Their orientations differ by some rotation. But a crystal looks the same from several directions — a hexagonal crystal from twelve, a cubic one from twenty-four — so “the rotation between them” is not one rotation. It is a whole family, all describing the same physical relationship. Standard practice picks the family member with the smallest angle and calls it the disorientation.

That picks the angle. It does not pick the rotation. Several family members share the smallest angle and differ only in their axis. If your measurement is the angle, you are safe. If your measurement multiplies rotations together — which every loop integral, circulation, and closure-failure calculation does — you have just multiplied a choice you did not know you were making.

2. THE AMBIGUITY IS TEXTBOOK. THE GAP IN THE CONVENTION IS NOT.

What follows is not new, and we want that on the page before anything else.

That a misorientation between two crystals has many symmetrically equivalent descriptions is standard material. For cubic symmetry there are 24 equivalent orientations on each side, hence **576 = 24 × 24** equivalent misorientations, conventionally reported in axis-angle form as 24 axes each carrying an angle, with the minimum-angle permutation used as the description. This appears in reference texts, in instrument-vendor documentation aimed at practitioners, and in the definition of the word *disorientation* itself.

Selecting a representative is also long-settled as a problem. Heinz and Neumann devoted a paper in *Acta Crystallographica* to representing orientation and disorientation data for cubic, hexagonal, tetragonal and orthorhombic crystals, and adopted an explicit convention so that the representation would be one-to-one. Anyone who works in this area knows the equivalence class is large and knows a convention is required. (*Volume, year and pages for this reference are pending a publisher-record pass; see Deposit Metadata.*)

Here is the narrow gap this paper is about. The everyday convention — *take the minimum angle* — determines the angle uniquely. It does **not** determine the rotor. The minimum is attained by $|G|$ distinct rotations at once, sharing an angle and differing in axis, and which one a program returns is decided by the tie-break inside its `argmax`. We have not found that stated anywhere, and its consequences are what Section 3 measures.

We also state plainly what this paper is *not* claiming, because a related literature exists and does not cover this. EBSD round-robin exercises have repeatedly shown that different laboratories obtain substantially different results from nominally identical samples — the ASTM E2627 round robin and the international grain-size exercises are the standard references — and that variance is attributed to **acquisition**: calibration, indexing quality, hit rate, step size, operator choices. Everything measured in Section 3 sits *downstream* of all of that. **Same file, same bytes, same code, two computers.** No acquisition difference can explain it.

We report the identity below first because reporting it as an empirical discovery about magnesium would be wrong, and we made that mistake before catching it.

For a misorientation m and point group G , the symmetry-equivalent candidates are the two-sided products $g_1 m g_2$. The scalar part of a quaternion product is cyclic — $\text{Re}(xy) = \text{Re}(yx)$ — so

$$\text{Re}(g_1 \cdot m \cdot g_2) = \text{Re}((g_2 \cdot g_1) \cdot m)$$

The scalar part therefore depends only on the product $g_2 g_1$, which takes $|G|$ values. Each is shared by exactly $|G|$ of the $|G|^2$ candidates.

Measured, three ways:

- the identity holds to 1×10^{-14} across all 144 candidate pairs for 622
- 2000 uniformly random rotations: a twelve-way tie in **100.00%** of cases, with a median of 12 distinct scalar-part values per rotor
- on a real map: 99.96%, the shortfall being floating-point noise splitting a true tie at the rounding tolerance

And the tied candidates are genuinely different rotations. Across 300 random misorientations we find **12 distinct rotors in 100% of cases** — identical angle to 1×10^{-13} , axes forming the full point-group orbit. A worked example: twelve rotors all at 15.211864° , axes spanning $[-0.9914 \ -0.0086 \ 0.1303]$ through $[0.9914 \ 0.0086 \ 0.1303]$.

The tie is textbook. **The consequence is what nobody measures.**

3. WHAT IT COSTS, MEASURED

All numbers below come from `twins.ctf` — the MTEX magnesium twin sample, 167×137 pixels at $0.3 \mu\text{m}$, square acquisition grid, identity verified against the depositor’s own hash before any analysis.

Between two machines, on identical bytes

quantity	value
plaquette cells compared	22,485
NaN (unreadable) pattern identical	yes
bitwise identical	18,013
differing by more than 1°	453
changing defect classification	201
largest single-cell difference	177.31°

One cell reads **0.0000° on one platform and 146.6018° on the other**. The cause is a one-ulp difference in library `sin/cos` between operating systems, which flips which tied representative an `argmax` returns.

Across five valid tie-break rules, one machine

tie rule	seam count	residue median	residue p95
first index	951	60.0000°	63.51°
last index	975	59.78°	66.60°
random, seed 0	2205	59.32°	77.85°
random, seed 1	2197	59.57°	79.23°
random, seed 2	2207	59.35°	77.26°

The seam count spans a factor of 2.3. Every rule is defensible. None is more correct than another.

Grain-ring closures

Rectangular loops enclosing each grain, under four gauges:

- per-loop movement: **median 67.4°**, maximum 158.7°
- 96%** of loops move by more than 10°
- rank correlation against the shipped gauge: **0.022, 0.223, 0.319**

A quantity whose rank order does not survive its own convention is not a measurement. We report ours as one that did not.

Three composed rotors around a triple junction

Constructed junctions whose **true closure is exactly zero** — a perfect I-junction with no rotational incompatibility — so any spread is pure representative freedom. Cubic 432, sweeping all $24^3 = 13,824$ combinations:

boundary misorientation	closure spread, median
2-15° (low-angle)	28.74°
15-30°	73.94°
30-62° (high-angle)	120.73°

Even three composed rotors carry tens of degrees of freedom on a junction whose correct answer is zero.

4. WHAT SURVIVES: THE INVARIANT TIER

Not everything moves, and the split is sharp. Every quantity built from angles reproduces **bit-exact** across both platforms:

quantity	value
boundary steps	3269
median angle in the $86.3 \pm 3^\circ$ window	86.1786°
axis distance to nearest 2-fold	0.2592°
axis tilt from basal	0.2744°
fraction of boundary steps in window	0.5390

The reduction margin, derived rather than quoted. The Mg {10-12} twin written every way the point group allows:

86.300	93.700	101.631	107.361	137.212	140.008	180.000
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Seven descriptions of one object. The smallest is 86.300, the runner-up is 93.700, and the gap is **7.400°** exactly. That is a fact about hexagonal symmetry — a better detector moves it by zero. Against typical Hough noise of 0.5° it is **SNR 14.80**.

The detector floor is a threshold, not a slope. A single-crystal synthetic map, true charge identically zero, perturbed by Gaussian noise of width σ :

σ	0.006°	0.057°	0.5°	2.0°	10°	30°
false charge	0.0000%	0.0000%	0.0000%	0.0000%	28.63%	85.74%

Three seeds each, no spread worth reporting. **Exactly zero false alarms across a 300-fold increase in noise, then a cliff.**

5. THE FIX, AND A THIRD-PLATFORM CHECK

Three changes, all small:

1. **Build the group elements from exact constants.** For 622 that is 0, $\pm\frac{1}{2}$, $\pm\sqrt{3}/2$, 1. Correctly-rounded square roots are identical on every platform; library trigonometry is not.
2. **Quantise the selection key** before the `argmax` — round the scalar part at 10^{-12} , so floating-point noise cannot flip the pick — and break remaining ties by a fixed index.
3. **Split the record.** Publish an *invariant-tier* hash over the class-invariant quantities and a separate *gauge-tier* hash over the representative-dependent ones. Cross-machine agreement on the invariant tier is the bar. Agreement on the gauge tier is a bonus.

Verification on a third machine. The hardened pipeline reproduces the seam count exactly:

	seam count
platform A, unhardened	1716
platform B, unhardened	1719
platform C, hardened	951
prior record	1716–1719 → 951

Exact to the integer, on a machine that had never run it.

6. THE ROUTE THAT IS ALREADY IMMUNE

Not every disclination method inherits this problem, and the one that does not is worth naming, because the reason has not been stated.

Zhu et al. (*Acta Materialia* **264**, 119600, 2024) calculate triple-junction disclination strengths from EBSD maps by combining Bollmann’s theory with a polytopal cell-complex representation of the microstructure. Their Section 3 opens by noting that the misorientations themselves cannot characterise a triple junction, because taking them all against one reference frame forces their product to the identity.

That is the same algebraic fact we measure in Section 4 of our own pipeline: a single-valued orientation field makes every loop close to zero exactly. We measured **1.8×10^{-5} degrees** on grain-consistent gauged loops, and an earlier machinery audit measured **7.04×10^{-6} degrees** over 96-step loops. It is a good identity to have verified independently, and it is their premise, not a finding.

Their solution does not compose reduced rotors at all. Bollmann’s O-lattice route transforms each misorientation into crystal-lattice coordinates through the structure matrix, decomposes it into integer and fractional parts, selects nearest-neighbour unit-cell corners under the lattice metric, and builds a unimodular matrix whose inverse describes the boundary’s dislocation structure. The junction quantity is a product of those **integer** matrices, and the strength is a rank-determinant of it — which is why their published strengths are integers rather than decimals.

So the twelve-answers problem does not reach them. The objects being multiplied are not symmetry representatives. That is a structural advantage of the O-lattice route over minimum-angle reduction, and we have not seen it written down. It is the practical recommendation of this paper: **if your quantity composes anything around a loop, do not compose reduced rotors.**

One narrow question remains open on their side and we flag it without claiming it is a defect: their construction contains a discrete fallback, selecting a larger nearest-neighbour distance when the smallest one yields a degenerate determinant. Whether the reported strengths are stable under that selection is not reported. It is one paragraph of work for whoever holds the code.

7. WHAT THIS DOES NOT CONTAMINATE

The most-published EBSD defect quantity is geometrically necessary dislocation density, computed from a curl of the orientation field — structurally the same loop integral. We expected it to inherit the problem. It does not.

tie rule	median GND density	change
first (shipped)	$1.3942 \times 10^2 \text{ m}^{-2}$	—
random, seed 0	$1.3948 \times 10^2 \text{ m}^{-2}$	+0.04%
random, seed 1	$1.3947 \times 10^2 \text{ m}^{-2}$	+0.03%
random, seed 2	$1.3948 \times 10^2 \text{ m}^{-2}$	+0.04%

Maximum deviation 0.04%, against a 5% bar committed before the run. GND density is robust to the representative choice. Kernel average misorientation, being an angle average, is immune by construction.

This is the most useful negative in the paper. The quantity most people compute is safe. The exposure is in composed-loop quantities specifically.

8. THE TEST, IN ONE PAGE

For anyone with a pipeline and no appetite for the theory:

1. Run your analysis. Save every number.
2. Change one thing: the rule that breaks ties in your symmetry reduction. First index $\rightarrow$ last index is enough. If you cannot find such a rule in your code, you are using one anyway; it is your language's `argmax`.
3. Re-run. Diff.
4. Any number that moved is gauge. Publish it with the gauge named, or do not publish it.
5. Permute the order of your symmetry operator list and repeat. Same test, independent failure mode.

Numbers that survive both go in the invariant tier. That tier is what should carry a cross-machine hash.

9. LIMITS, AND THREE OF OUR OWN RESULTS THAT DIED

One map, one material. Everything measured here is hexagonal magnesium at 0.3 μm step. The theorem is general; the magnitudes are not. Cubic materials have twenty-four representatives rather than twelve and should be worse, but we have not measured that on real data.

No competence surface. The axes that govern this — step size relative to grain size, and angular noise — are not mapped. That is the obvious next work and we have not done it.

Three results we killed:

- **A spike at exactly 60.000000°** in the residue distribution, present in 11.6% of cells against **zero hits in 500,000 null draws**. It looked like quantisation on a point-group element. It moves to $\sim 1\%$ under any other tie-break rule and under seven of eight symmetry-operator permutations. It was manufactured by the first-index rule.
- **A defect census of the grain network**, counting cycles where a consistent gauge cannot be built — which turns out to be Bollmann's fifty-year-old U-junction/I-junction classification rediscovered. Ours was also spanning-tree dependent: the frustrated fraction ranges 62% to 83% across eight trees on the same map. Not an invariant.
- **A null on the composed-loop residue** against a random-rotation baseline, the composed-loop analogue of the Mackenzie distribution. The measured median sits $2.2\text{--}2.9^\circ$ below the null under every gauge — above the instrument's noise floor, below our pre-registered 5° separation bar. **Reported inconclusive, and the bar was not moved.**

Each was caught by a control written before the run. That is the only reason they are in Section 9 rather than Section 3.

DEPOSIT METADATA

Data. `twins.ctf`, MTEX sample data, sha256 `cdb0e40625331790803c9c67f6ab56a1908c1997700c13e4bf21f773427171ca`, verified against the depositor's record before analysis.

Code. All scripts ship with the deposit. Every number in this paper appears in a tool output; nothing lives only in prose.

Established, cited, not claimed. The multiplicity of symmetrically equivalent misorientations (576 for cubic) and the minimum-angle disorientation convention; the selection of a unique representative for disorientation data (Heinz & Neumann, *Acta Cryst.*); EBSD round-robin interlaboratory variability and the associated grain-size standards including ASTM E2627; Bollmann's O-lattice construction and the U/I junction classification; the Mackenzie distribution; disclination density tensors from EBSD; the polytopal cell-complex TJD method (Zhu et al., *Acta Materialia* 264, 119600, 2024).

Asserted only as "not located." That the minimum-angle criterion is itself $|G|$ -fold degenerate in the rotor; that the degeneracy propagates through composed-loop quantities; and the invariant-tier / gauge-tier hashing discipline.

Prior-art status. No credentialed database pass has been run. The search covered publisher pages, arXiv and repository records. Novelty language is restricted to "not located," never "new." The Bollmann O-lattice framework, the U/I junction classification, the Mackenzie distribution, disclination density tensors from EBSD, and the polytopal cell-complex TJD method are all established prior work and are cited as such.

Citation verification. The Zhu et al. reference is verified against the article's own header. Remaining references are pending a publisher-record pass and are marked as such in the reference list.

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