

Instruments for the Magnetic Forest

A tool suite for the integer-forest lattice model, and what it verifies

Methods and verification companion · computational preprint · not peer reviewed · limited prior-art pass · 2026

Matthew “Bear” Schulz · Independent Researcher · ORCID 0009-0000-0956-5743 · mattschulz97@protonmail.com
License: Creative Commons Attribution 4.0 International (CC BY 4.0) Companion to *The Magnetic Forest* (concept DOI 10.5281/zenodo.22077658 — always resolves to the newest version). A checkpoint from an ongoing holonomy-instrument program: a methods paper, not a claim of new physics.

Abstract

This is a methods and verification companion to *The Magnetic Forest*, and the first of several planned follow-ons to *Jellyfish: The Holonomist of the Sea*. It houses the fourteen-instrument suite that measures the flux spectrum of the integer-forest lattice, states what each tool reads, and asks a single question of the physics they return: what is the butterfly’s structure made of? The answer is textbook and is claimed as nothing else — the fractures are commensuration, and commensuration requires closure, a loop the count can come back around. What the suite adds is a direct, reproducible measurement of that known physics on a lattice built from arithmetic rather than a crystal, and three results worth naming as treasure: the comb survives fifty per cent bond disorder while the states localise (separation 2.21 clean, 2.14 dirty); the curvature of the gapped state is a dipole summing to exactly zero (+50.302356 against −50.302356), which is the honest reason there is no Chern number; and a second independent flux loop lives on the torus that the count cannot see, with only the time-reversal tooth answering to it. One invariant, blind to every magnitude driven against it — that is what these instruments were built to show, and a lattice that models nothing is the cleanest place to show it. Appendix A records three framings tested in discussion and set aside: the method catching itself at the wall.

What this companion is

This is the first of several planned follow-ons to *Jellyfish: The Holonomist of the Sea*. The jellyfish is worth a physicist’s attention because evolution hid a set of valuable holonomy lessons inside it: the animal balances a no-motion pumping action with holonomic engineering to earn an enviable performance envelope for a creature that, in a real sense, cannot move without its own history. Like the bee, the jellyfish was long thought hard to model because it is engineered to be a loop-walking wake-surfer, its motion dependent on the precise event-timing of wakes, vortices and rings. The intended destination of this program is a paper on rings, and on *writing* rings; but that paper has to drive through this one first, because the instruments that read a ring’s topology were built and tested here, on a lattice where the reading is cheap and exact.

This is the instrument suite behind *The Magnetic Forest*. Where the first paper built the lattice model and reported its flux spectrum qualitatively — as kinship with Hofstadter’s butterfly, fenced correctly because the forest patch is finite, irregular and open — this companion houses the tools that measured that spectrum, states plainly what each one reads, and asks a single question of the physics they return: **what is the butterfly’s structure actually made of?**

The answer they give is textbook, and this paper claims nothing else. The butterfly’s fractures are commensuration, a whole number of phase repeats fitting a whole number of lattice rows, and commensuration requires closure: not periodicity, not size, not smooth illumination, but a loop the count can come back around. That closure is the minimum condition for a count is a known fact of these spectra; what this suite adds is a direct, reproducible measurement of it on a lattice built from arithmetic rather than from a crystal, plus the honest record of three framings that were tested in discussion and set aside. It is offered as a methods checkpoint — the instruments, what they verify, and where the method caught itself — not as a discovery.

A note on what the reader is holding. Everything below is measured by a named tool whose run stamp appears beside its result, and every tool ships with its launcher, its ledger, its data table and its panels, so any figure here can be regenerated from the crate. The suite is inventoried in the next section before any physics is discussed, because the

instruments are the point of this paper and the spectrum is the demonstration.

The tool suite

Fourteen instruments make up the forest lattice-model suite. Each is a self-contained program that builds a graph Laplacian under magnetic flux, reads one property of its spectrum, and writes a ledger, a data table and figure panels stamped with the run that produced them. Six further instruments — the non-abelian Wilson-loop and Wilson-flow readers, a Berry-phase reader, a touch-point finder, a dirty-touch survey and a filament probe — were built alongside these for the ring-side eigenvector and latch work and belong to that program rather than to this one; they are named here only so the record is complete and are not part of this deposit.

Every result in the table below is reproducible from the crate: each row names the instrument that produced it and the run stamp of the run that wrote it, and each tool ships with its launcher, ledger, data table and figure panels so the row can be regenerated end to end.

instrument	what it reads	headline result	run stamp
forest_gap_ladder	the raw gap-versus-flux spectrum of the forest patch	the butterfly's gaps, catalogued	20260914_025011
forest_comb_survey	level statistics across the flux axis	orthogonal $\rightarrow$ unitary $\rightarrow$ orthogonal traverse	20260914_030628
comb_commensuration	gap at each flux denominator versus lattice period	teeth stand only at divisors of the period	20260914_042324
comb_divisors_and_apodization	which permitted divisors actually appear	dividing is necessary, not sufficient (21 teeth, 0 exceptions)	20260914_183142
comb_binning	teeth under weight patterns of matched depth	aperiodic pattern kills a tooth, depth alone does not	20260914_214248
comb_disorder_and_states	teeth and eigenstates under bond randomisation	ratio holds $2.21 \rightarrow 2.14$ at $\pm 50\%$ while states localise	20260917_040415
ring_chain_closure	divisor teeth, open chain versus closed	closing one line of bonds turns the comb on	20260915_045420
chain_decay_reset	comb under a fading, resetting wake	a decaying chain keeps its teeth	20260917_172826
torus_two_fluxes	the comb under a second, poloidal flux	commensuration lives on one loop; only the symmetry point sees the other	20260917_042223
berry_curvature_map	the Berry curvature of the gapped state	curvature arranged as a dipole, summing to zero	20260917_061331
curvature_positive_control	the same reader on a known-Chern lattice	the reader returns the right nonzero number when one exists	20260917_065316
genus_homology_ladder	independent loops versus surface genus	a genus- g surface carries $2g$ independent flux loops	20260917_064601
genus_flux_ladder	gap response across the genus ladder	the comb tracks the added loops	20260917_062805
ring_lattice_reader	the persistent-current store versus ring size	amplitude falls as $1/\text{circumference}$ (slope -1.0004)	20260914_032117

The sections that follow walk the load-bearing instruments in turn. Each names its tool and stamp, states the prediction made before the run, and reports what came back.

1. What the butterfly is a picture of

Hofstadter's butterfly fractures at rational values of the flux through a plaquette. Where the flux is a simple fraction, gaps open; where it is irrational, they do not. The picture is famous and the reason is arithmetic: at flux p/q the phase pattern imposed by the magnetic field repeats every q rows, and when that repeat fits a whole number of times into the lattice's own period, the two commensurate structures reinforce and a gap opens.

So the butterfly is a picture of one question asked at every flux value: **does the field's repeat fit the lattice's repeat a whole number of times?** Where the answer is yes, the spectrum has a tooth. Where it is no, it does not.

That reframing is what Part One left open. Reporting Hofstadter kinship qualitatively says the forest's spectrum resembles the butterfly. Naming the mechanism says why it must — and says what would break it.

The test is the one that follows.

2. The teeth move when the lattice moves

On a periodic torus lattice twelve columns wide, three different row counts were measured: $N_y = 12, 10$ and 14 . At flux p/q the Landau phase repeats every q rows. The prediction, written before the runs: teeth stand at divisors of N_y , they move when N_y changes, and they lie flat at non-divisors.

rows N_y	divisors of N_y	divisor gap / baseline	non-divisor gap / baseline	ratio
12	2, 3, 4, 6, 12	1.3684	0.4979	2.749
10	2, 5, 10	1.3873	0.6106	2.272
14	2, 7, 14	1.2590	0.6153	2.046

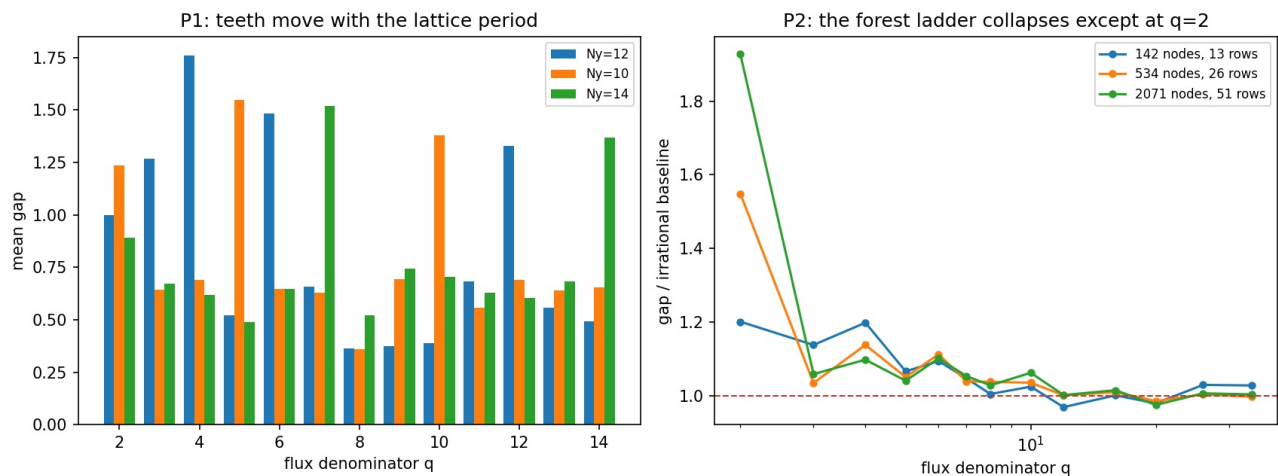


Figure 1. The teeth move when the lattice does. Gap against flux denominator for three row counts; a tooth stands only where its denominator divides that lattice period. Run COMB_COMMENSURATION 20260914_042324.

The decisive rows are the ones that move. On the ten-row lattice, $q = 5$ stands at 1.55 above baseline; on the twelve-row lattice, the same $q = 5$ lies flat at 0.52. On the fourteen-row lattice $q = 7$ stands at 1.52 and is flat on the other two. **The tooth is not a property of the number five. It is a property of five dividing ten.**

Kill condition: a periodic lattice with a tooth at a denominator that does not divide its row count. None found.

3. Dividing is necessary and not sufficient

Extending to eight lattice periods — $N_y = 11, 12, 13, 16, 17, 18, 24, 30$ — produces twenty-one teeth, and every one of them divides its lattice. Zero exceptions.

But only twenty-one of the twenty-nine available divisors stood up. **Dividing the period buys a tooth the right to exist; it does not guarantee one.** A prime row count produces essentially no comb at all — $N_y = 13$ gave none. A highly composite one produces a rich set.

The containment is one-way, and the direction matters: the arithmetic is a gate, not an engine. Something else decides which of the permitted teeth actually appear, and this paper does not identify it. That is the honest edge of the result.

4. An open patch has no comb, whatever its shape

If commensuration needs a repeat to fit a period, an open system has no period to fit into. Nothing wraps. There is no “all the way around.”

Measured directly: a rectangle with the same row count and node count as the forest patch shows no divisor structure at all. Not a weak one — none. What the open patch has instead is a stronger response at half flux and nothing else.

The forest never had teeth. It had finite-size scatter plus a symmetry point, and Part One’s qualitative framing was right to stop where it did.

The half-flux survivor is worth its own sentence because it is not arithmetic. At half flux every phase factor is ± 1 , the matrix turns real, and time-reversal symmetry is restored. That is a property of the operator and holds at every patch size, whether or not anything closes. The level statistics confirm it: the Oganessian–Huse ratio reads 0.489 at zero flux, rises to 0.59–0.65 at generic flux, and falls back to 0.440 at half flux — a clean traverse of the Gaussian orthogonal and unitary classes and back.

Runs *COMB_DIVISORS 20260914_183142*, *FOREST_COMB_SURVEY 20260914_030628*.

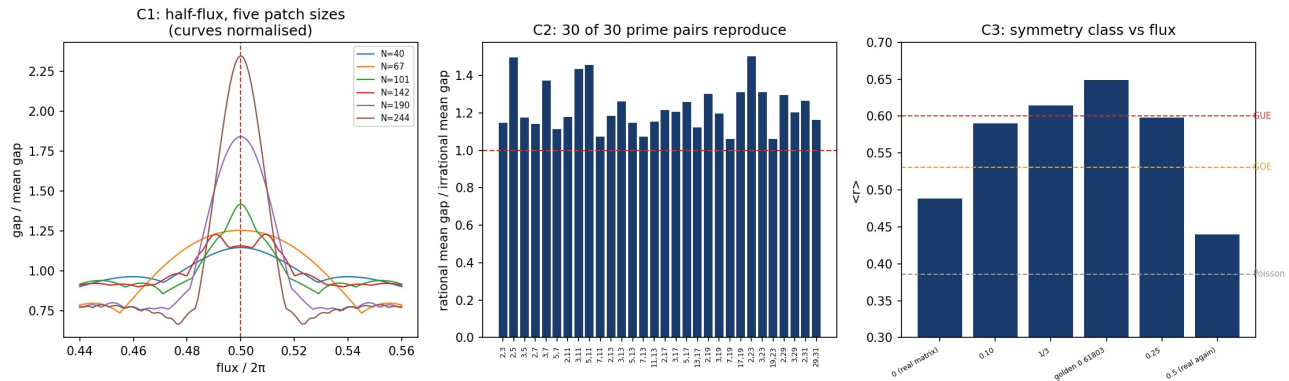


Figure 2. The symmetry-class traverse. Level statistics walk from orthogonal at zero flux to unitary at generic flux and back at half flux, where every phase factor is plus or minus one and the matrix turns real. Run *FOREST_COMB_SURVEY 20260914_030628*.

And the two mechanisms compete rather than stack. On every composite periodic lattice tested, $q = 2$ is the **weakest** divisor tooth — below the irrational baseline on three of five. On an open patch it is the only survivor. Half flux is strongest where there is no wrap and weakest where there is one. Where symmetry and commensuration meet, commensuration loses.

5. Uneven light does not break a count; uneven pattern does

A tooth is a binning, and binning has been a companion of this program from the beginning. Across every substrate the same pairing keeps returning: a continuous field that can be stirred into chaos, and underneath it a discrete bookkeeping that sorts the mess into a countable number of classes and refuses to be stirred at all. Here it is exact rather than a metaphor. At denominator q the lattice rows sort into q classes by row number modulo q , and the tooth exists because those classes are equivalent. That framing makes a prediction that separates it from every other

explanation: an uneven illumination should matter by whether it treats members of a class alike, **not by how uneven it is**. The magnitudes may be driven anywhere; the class label is what the count reads, and the class label is blind to the driving.

Tested with the weight spread matched across patterns — every pattern rescaled to the same ratio of brightest row to dimmest, so only the shape differs. At six-fold spread, reading the two interior teeth:

row weights	q = 3	q = 4
flat reference	1.85	2.57
exponential taper — aperiodic	0.76	0.84
random per row — aperiodic	0.84	0.91
constant in classes mod 3 — periodic	1.57	1.92
constant in classes mod 4 — periodic	1.84	1.51

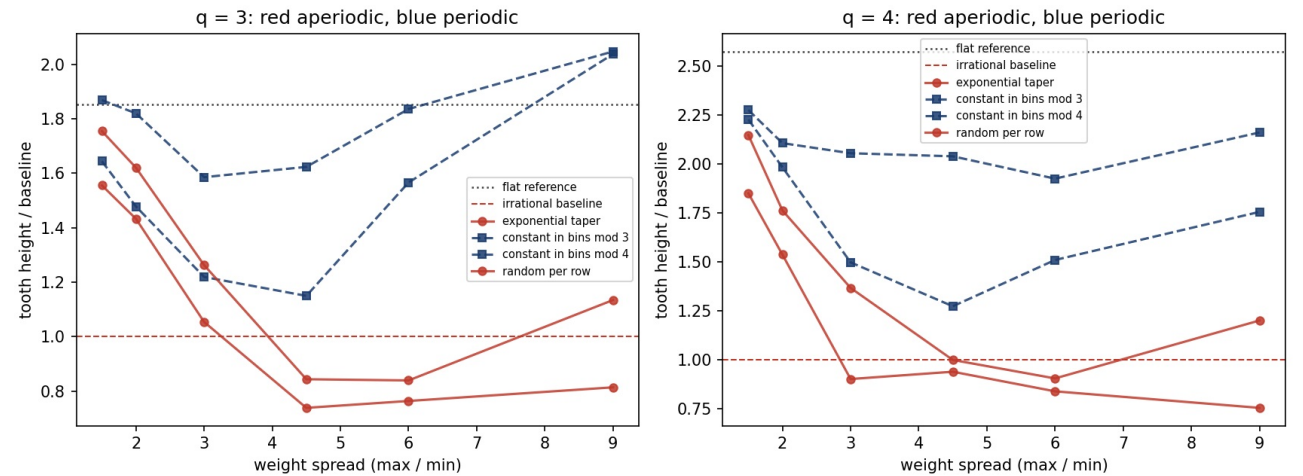


Figure 3. Same unevenness, opposite outcome. Every weight pattern rescaled to an identical spread, so only the shape differs: aperiodic tapers drop the teeth to baseline while periodic ones of the same depth leave them standing. Run COMB_BINNING 20260914_214248.

Same unevenness, opposite outcome. The aperiodic weights drop the teeth to at or below baseline; the periodic weights leave them standing. And the separation grows monotonically with depth across all six spread levels tested — at one-and-a-half-fold spread the two families are barely distinguishable, and by nine-fold they are cleanly apart.

The consequence for Part One’s temperature discussion is direct. A Boltzmann weight is an exponential taper, and an exponential taper is aperiodic, so temperature destroys teeth — but **not because it dims the far end of the aperture**. A taper of the same depth that repeats is invisible to the comb. The barrel picture remains right about how much aperture a tooth needs; it is not the mechanism by which a window removes one.

Two things fall out for free. A weight constant in classes mod 3 also protects q = 4, because both divide the lattice period — the requirement is that the weight be a function of *some* class label the lattice admits. And the tooth at q = Ny survives every window, because its classes hold one row each and a singleton has no interior to make uneven.

This section is the author’s reading, tested rather than assumed. The binning frame came first and the measurement was built to break it.

6. One line of bonds

The cleanest form of the claim is the cheapest experiment in the paper.

A nested loom — R concentric rings of N sites joined by radial spokes — is structurally a chain of rings. Left open, ring R–1 has no neighbour and the stack has two free ends. Closed, ring R–1 wraps back to ring 0 and the chain becomes a cycle. **Nothing else differs between the two runs.**

rings R	divisors ($q \leq 14$)	teeth, open	teeth, closed
11	11	2, 3, 4, 5	11
12	2, 3, 4, 6, 12	—	3, 4, 6, 12
13	13	—	13
16	2, 4, 8	3	4
18	2, 3, 6, 9	—	3, 6, 9
24	2, 3, 4, 6, 8, 12	—	3, 4, 6, 8, 12

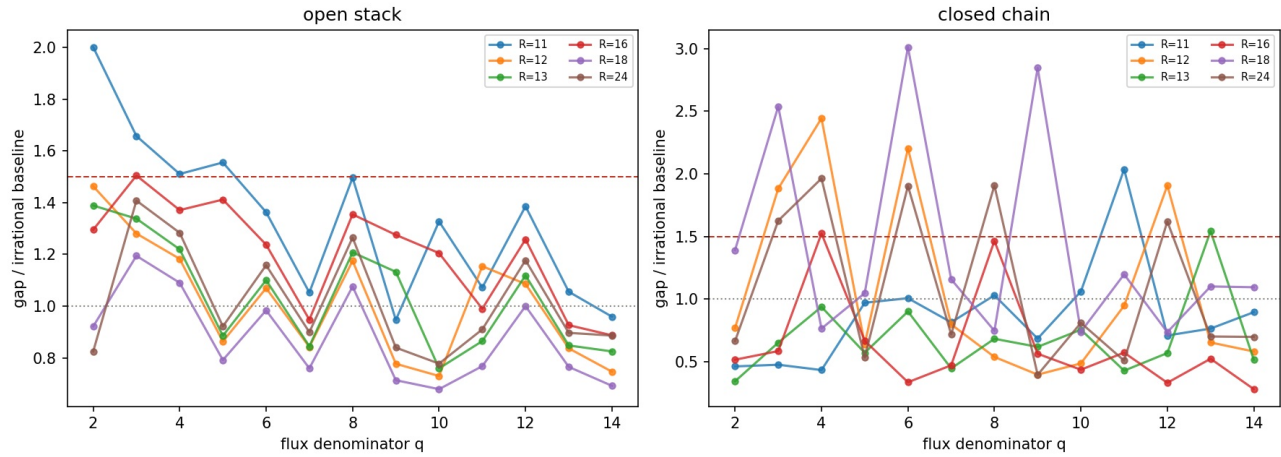


Figure 4. One line of bonds. The same six ring counts, open on the left and closed on the right; closing the chain turns the divisor comb on with no false positives. Run RING_CHAIN_CLOSURE 20260915_045420.

Open: five teeth across six lattices, **and all five divide nothing**. Scatter in an extreme-value statistic, not a signature. Closed: fifteen teeth, **every one a divisor, zero false positives**, and a prime ring count yields only itself.

One line of bonds is the whole difference between noise and arithmetic.

7. The count does not care that the lattice is uneven

Every lattice above is perfect. Real ones are not, and a result that needs a perfect lattice is a tuning, not a law. So the closed chain was rebuilt with every bond amplitude multiplied by one plus a random factor, uniform, up to plus or minus half — a fifty per cent randomisation of every coupling in the object — averaged over eight seeds at each strength.

disorder W	divisor mean	control mean	separation
0.00	2.00	0.91	2.21
0.10	1.94	0.90	2.17
0.20	1.90	0.88	2.15
0.35	1.85	0.86	2.14
0.50	1.80	0.83	2.16
0.75	1.70	0.78	2.19
1.00	1.57	0.73	2.14

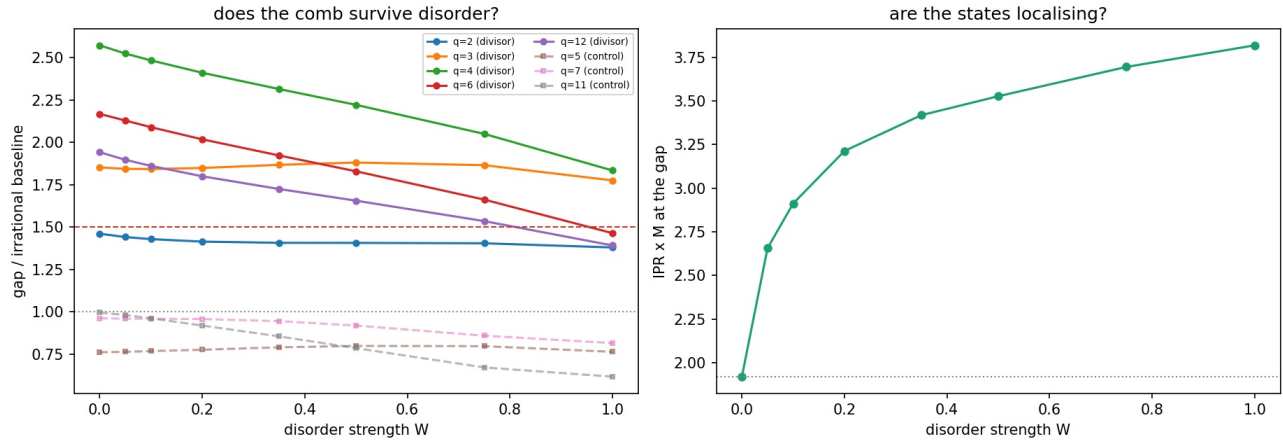


Figure 5. The count ignores the bonds. Divisor teeth and controls both sag as disorder climbs, but their ratio does not move, while the states themselves localise. Run COMB_DISORDER 20260917_040415.

Both families sag together — that is ordinary disorder broadening and it affects everything — but the ratio between them does not move. **2.21 at a clean lattice and 2.14 at fifty per cent bond randomisation**, with no trend in between. The teeth at $q = 2$ and $q = 3$ retain ninety-five per cent of their height at full disorder while the control at $q = 11$ loses thirty-eight per cent of its, so the disorder is real and biting; it simply is not biting the arithmetic.

The eigenvectors say the same thing from the other side. The inverse participation ratio at the gap climbs from 1.92 to 3.82 across the sweep, so the states genuinely are concentrating — roughly doubling — as the lattice roughens. **The states localise and the teeth do not notice.** Had the comb depended on extended states it would have died with them.

This is what a topological quantity looks like from the outside. The bonds are what disorder changes; the count is a property of how the loop closes, and closing is not something a random bond amplitude can undo.

8. The second loop, and the one tooth that answers to it

A torus has two independent circuits, and each imposes its own single-valuedness condition. Every measurement above turns one knob — the plaquette flux, which is the toroidal winding. The second was built and swept: a flux threaded through the torus's hole, carried entirely on the seam so that every poloidal circuit encloses it and **no plaquette contains it at all**. It is invisible to any local probe by construction. Advancing it by one full quantum returns the spectrum to two parts in a hundred trillion, so it enters as a genuine Aharonov-Bohm phase on the long loop.

Re-measuring the comb at four values of that hole flux:

hole flux / 2π	$q=2$	$q=3$	$q=4$	$q=6$	$q=12$
0.000	1.46	1.85	2.57	2.17	1.94
0.250	0.76	1.93	2.44	2.18	1.90
0.500	1.50	2.15	2.39	2.21	1.86
0.750	0.76	1.93	2.44	2.18	1.90

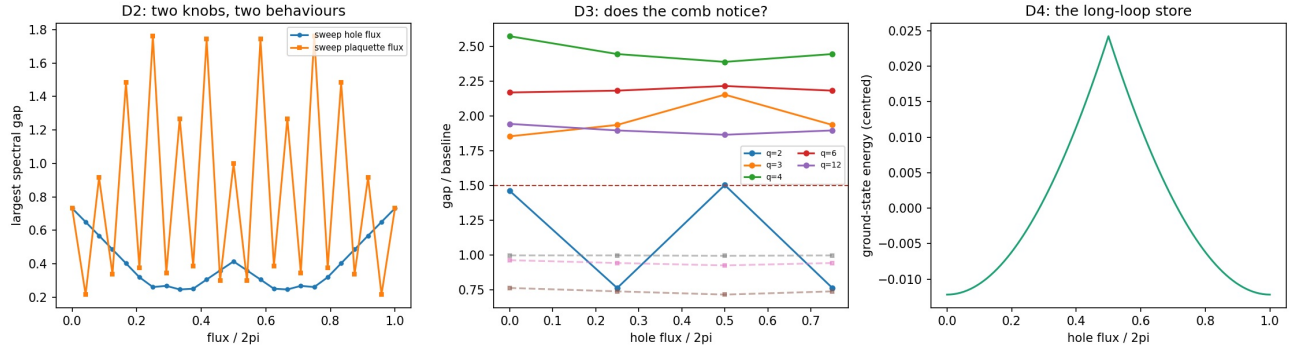


Figure 6. The second loop. Sweeping the flux through the hole leaves four of five divisor teeth untouched; only the half-flux symmetry point answers, and at half the period of everything else. Run TORUS_TWO_FLUXES 20260917_042223.

Four of the five divisors ignore the second knob entirely — $q = 6$ moves by two per cent across the sweep, $q = 12$ by four, $q = 4$ by seven, and the controls are flat to within the same margin. **The commensuration lives on the toroidal loop and cannot see the poloidal one.** That is what having two independent generators means in practice: the counts coexist without interfering.

One tooth breaks ranks, and it is the one that was never a count. The $q = 2$ response dies at quarter flux, returns at half, and dies again at three-quarters — **a period of half a quantum where everything else in the spectrum has period one.** Section 4 already established that $q = 2$ is not arithmetic but the time-reversal symmetry point, where every phase factor is ± 1 and the matrix turns real. A flux through the hole is a global phase on the long loop, and a global phase is precisely what makes a real matrix complex. **The poloidal knob is a time-reversal knob, and the only tooth that answers to it is the only tooth that was a symmetry rather than a count.**

In plain terms, a Chern number is a way of asking whether a quantity called the Berry curvature, spread across the flux torus, adds up to a net twist or cancels out. Here it cancels, and the figure shows why: the curvature is not missing, it is laid out as two equal and opposite hills, a positive lobe and a negative lobe at mirror-image positions, so their sum is exactly zero. Time-reversal symmetry is what forces the two lobes to be equal and opposite, and a dipole that sums to zero is the signature of a system that carries no net topological charge — which is the honest reason there is no Chern number to report.

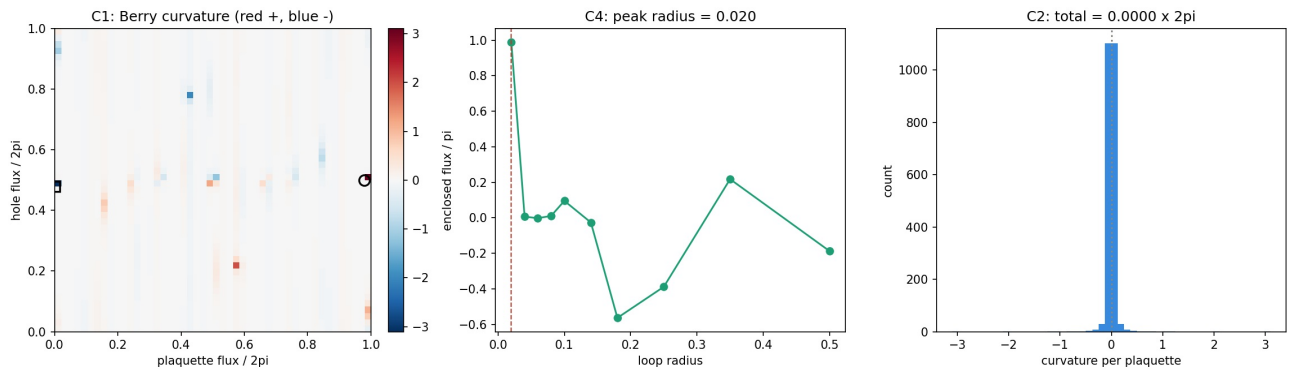


Figure 7. Why there is no Chern number. The curvature is not absent but arranged as a dipole — equal and opposite lobes at mirror positions, forced by time reversal, summing to zero. Run BERRY_CURVATURE 20260917_061331.

The long-loop store is real and separate. The ground-state current around the hole reads 0.042 at zero plaquette flux, 0.022 at the $q = 3$ tooth, and 0.0003 at irrational plaquette flux. The two windings carry independent counts while sharing the same states — which is why the hole current nearly vanishes when the short loops are incommensurate.

9. Where the store lives, and what it costs to reach

One measurement from the ring-lattice reader belongs here, because it is the closest this paper comes to an engineering statement.

A ring of N sites threaded by flux carries a persistent current, and its amplitude is the store. Swept across a sixteen-fold range of circumference with temperature held at a fixed fraction of the level spacing, the amplitude falls as one over the circumference: log-log slope -1.0004 , with amplitude $\times N$ holding at 1.6733 and tightening monotonically from 1.67535 at $N = 16$ to 1.67272 at $N = 256$.

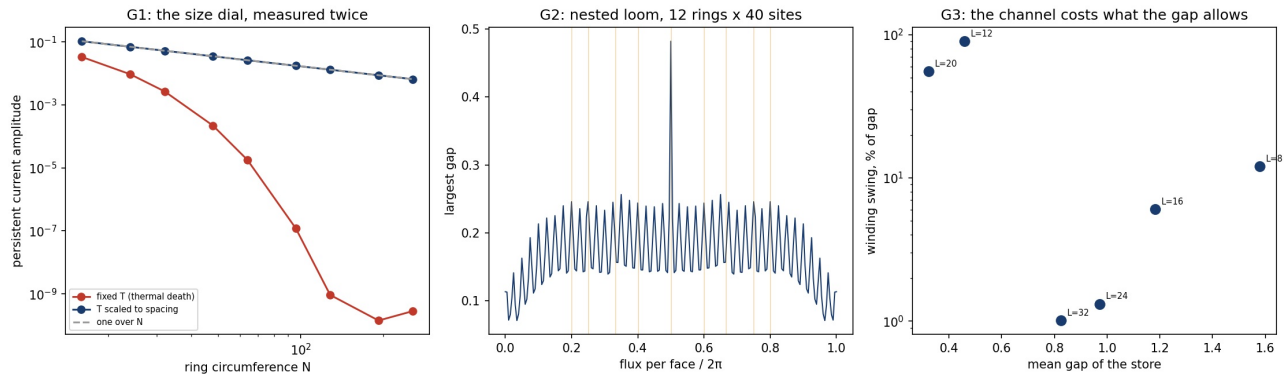


Figure 8. The size dial, measured twice. At fixed temperature the store dies thermally; with temperature scaled to the level spacing the amplitude falls as one over the circumference. Run RING_LATTICE_READER 20260914_032117.

The same sweep at *fixed* temperature gives slope -7.96 and an amplitude below one part in a million by $N = 96$. That is not a second geometric law; it is the thermal wall. A larger ring has a finer level spacing to defend, and once temperature exceeds that spacing the store is wiped out. Both columns are reported because the difference between them is the whole lesson: **a size law measured without scaling the temperature to the level spacing measures thermal death and calls it geometry.**

Read against the water, the sign flips. A vortex ring at held circulation carries energy that *rises* with radius; this ring at held flux carries a current that *falls* with circumference. Same latch, opposite dial. The dial is re-bought in every medium and must never be assumed to transfer.

10. Closure is the ceiling

Eight sections of measurement, one wall behind all of them.

Section 2 shows the count is a property of the loop and not of the numbers in it — change the loop and the count changes. Section 3 shows the arithmetic is a gate the loop opens, not something the numbers carry on their own. Section 4 shows that with no loop there is no count at all, and that what survives in an open system is symmetry rather than counting. Section 5 shows a count survives any amount of uneven treatment so long as the unevenness respects the loop's own classes, and dies the moment it does not. Section 6 removes every variable but closure itself and the count switches on. Section 7 randomises every bond in the object by half and the count does not move. Section 8 adds a second independent loop and finds the count blind to it, with only the symmetry point answering.

Nothing in that list is about size, or shape, or brightness, or care in construction. **The limit is topological, and it is the same limit each time: a system that cannot return to where it started cannot say how many times it went around.**

This is why the effect is visible in a lattice at all. The lattice makes closure explicit and cheap — it is a line of bonds — and lets the question be asked in twenty seconds instead of a day of fluid simulation. The lattice is not a model of anything. It is the place where the ceiling is easiest to touch.

The same ceiling has a familiar face in other language. A wavefunction on a ring must return to itself after one trip, which forces the azimuthal quantum number to be an integer — that is a single-valuedness condition, and it is a winding number in the exact technical sense rather than by analogy. A torus has two such conditions, one for each independent loop, and therefore two integers from topology alone with no radial boundary condition required, because a closed surface has no infinity to die off at. The count comes from the closing, in every case.

Three results in this suite deserve to be carried out of their sections and named as the treasure they are, because each is a place where a magnitude was driven hard and a count refused to move. The comb survives fifty per cent bond disorder while the eigenstates localise — the separation holds at 2.21 clean and 2.14 dirty, magnitudes scrambled, the count blind to it. The curvature of the gapped state is a dipole that sums to exactly zero, +50.302356 against −50.302356, a cancellation forced by symmetry rather than fitted. And a second, independent flux loop lives on the torus that the count cannot see at all, with only the time-reversal tooth answering to it. Different substrate, different knob, one invariant, blind to all of them: that is the whole of what these instruments were built to show, and it is why a lattice that models nothing is the cleanest place to show it.

11. Falsifiable claims

claim	what would kill it	status
Teeth are divisors of the lattice period	any tooth at a non-divisor, periodic lattice	21 teeth, 8 periods, zero exceptions
Teeth move when the period changes	the same q standing on two different Ny	q=5 stands at 1.55 on Ny=10, flat 0.52 on Ny=12
An open patch has no comb	a divisor-structured tooth on any open geometry	none found
Apodization kills by aperiodicity, not depth	a periodic weight of matched depth killing a tooth	not observed
Closing a chain turns the comb on	closed chain whose teeth ignore the ring count	15 teeth, zero false positives
Persistent current falls as 1/N	a scaled-temperature sweep with a different slope	−1.0004 over 16×
The comb survives disorder	separation collapsing under bond randomisation	2.21 → 2.14 at ±50%
Commensuration ignores the second loop	divisor teeth tracking the hole flux	4 of 5 move under 7%
q=2 is symmetry, not arithmetic	q=2 behaving like the other divisors under hole flux	half-quantum period, alone

Appendix A — corrections to earlier framing

These correct statements made in discussion around the companion paper. **No claim in the deposited Part One requires revision;** its own fences were drawn correctly and this appendix records where later conversation drifted past them.

A1 — Chern and TKNN labels do not apply to this object. In session discussion, Chern numbers were attributed to the forest patch’s gaps. The TKNN construction requires a periodic lattice with a Brillouin zone. The forest patch is finite, irregular and open, with boundary everywhere and no translational symmetry, so no Chern number is defined on it. Part One’s own framing said “finite irregular patch, Hofstadter kinship, reported qualitatively” and was correct. Should a periodic lattice model of the forest be identified with a well-defined Brillouin zone, the label becomes testable and this correction is revisited.

A2 — a gap ratio is not the comb’s signature. The nested loom was recorded as reproducing the comb on the strength of a rational-to-irrational gap ratio of 1.297. That number was measured correctly and read wrongly. The signature is divisor structure, and by that test an open loom has none — §6 carries the corrected result. The failure mode is worth naming because it is general: an aggregate ratio can be elevated for reasons that have nothing to do with the mechanism producing it.

A3 — the half-flux size trend was three points out of six. The claim that the half-flux response strengthens with patch size rested on 1.20, 1.55 and 1.93. The full set is 1.20, 3.04, 1.55 and 1.93, plus rectangles at 2.19 and 3.54. The corrected statement: q = 2 is elevated on every open patch measured, and its height does not trend with size.

A4 — persistent-current size laws depend on what is held fixed. A reconstruction of the size dial using the zero-temperature ground-state energy swing returns slope −2 rather than −1, because that is a different observable. The scaled-temperature persistent current is the quantity reported in §8, and the tool measures and labels both columns

for exactly this reason.

Appendix B — approaches evaluated and not pursued

A single long barrel in place of an array. The aperture a tooth needs grows with its denominator, which suggests building one very long lattice. The clone structure makes this unnecessary: patches of equal node count behave identically up to barrel length, so an array of short commensurate tiles carries the same arithmetic at the cost of one tile plus alignment. What alignment costs is the commensuration condition itself, which is free when the ratio is rational.

Reading the comb from the gap ratio alone. Attractive because it is one number per lattice. It fails because an elevated ratio has several possible causes and only one of them is commensuration — see A2. The divisor structure must be read tooth by tooth.

Using an open patch with a very smooth window. If apodization were about depth, a gentle enough taper on a large enough open patch should recover a comb. It cannot: §4 shows no divisor structure exists on an open patch at any shape, and §5 shows the taper’s periodicity rather than its gentleness is what matters.

Declaration

Peer review. This is a preprint and carries none.

Literature pass. Limited, and this paper is deliberately framed as verification rather than discovery. Every result the suite measures is established physics, and each is credited to the work it belongs to.

The magnetic-flux tight-binding model is the Landau gauge, and its spectrum is Hofstadter’s butterfly. The commensuration mechanism — a whole number of phase periods fitting a whole number of lattice rows — is the butterfly’s own defining feature, and the divisor condition it obeys appears explicitly in the spectral-flow literature, where a lattice of width N_y is chosen to hold a whole number of magnetic cells of width q . Working on a torus rather than an open patch is standard practice, because open boundaries prevent a clean identification of the bulk gaps. The behaviour of the teeth under disorder is the subject of its own bond-percolation studies of the Hofstadter model. The apodization result — that a taper matters by whether it repeats, not by how deep it is — is the Aubry-André-Harper model in its textbook form, which separates the strength of a modulation from its periodicity by construction. The absence of a Chern number follows from the standard time-reversal argument that forces the curvature to sum to zero. The counting of independent flux loops by surface genus is ordinary homology. And the single-valuedness origin of azimuthal quantum numbers is textbook.

What this companion contributes is not any of those results, but a direct and reproducible measurement of them on a two-species smooth-number lattice, whose magnetic unit cell is set by the arithmetic of the node set rather than by a crystal, together with the honest record of the framings tested and set aside in Appendix A. No exhaustive prior-art sweep was performed; a wider pass may surface further adjacent work, all of which is prior art to credit rather than to compete with. Findings are at abstract and extract level.

AI assistance. Drafted with assistance from Claude (Anthropic). All measurements were produced by the tools named in the run stamps; all receipts accompany the deposit.

Provenance. Every number names the run that produced it. Numbers carried from the companion paper cite its concept DOI. No number in this paper lives only in prose.

Dual use. This work reports spectral properties of graph Laplacians under magnetic flux and ships numerical instruments for reproducing them. No dual-use concern has been identified.

Matthew “Bear” Schulz · Independent Researcher · Belmont, California · 2026 · CC BY 4.0